UNIVERSITY OF GRONINGEN WMBE024-05

SEMESTER 1 ASSESSMENT PAPER 2025/2026

CONTROL ENGINEERING

FINAL EXAM

EXAM DATE: OCTOBER 31, 2025

COURSE COORDINATOR: PROF. M. GHANDCHI TEHRANI

DURATION: 2 hours

This paper contains two questions.

All answers must be in provided answer sheets.

Answer **ALL** questions.

An outline marking scheme is shown in a square to the right of each question.

Linear-Logarithmic Graph Paper is provided with this examination paper.

A table of Laplace Transforms and few formulas are given at the end of this examination paper.

Only University approved calculators may be used.

Question 1

20 Marks

Part a

Consider the electrical circuit shown in Figure 1, where $v_o(t)$ represents the voltage across the resistor and $v_i(t)$ is the input voltage. The component values are given as $L_1 = L_2 = 1$ H, $C = 1$ F, and $R = 2\ \Omega$.

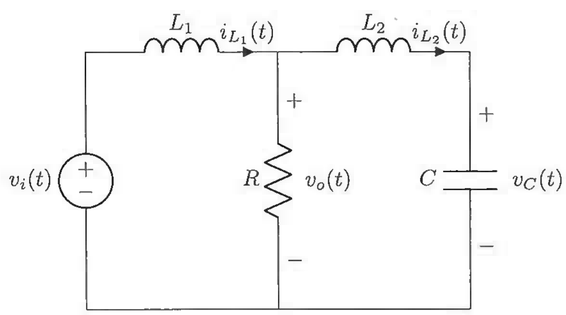

Figure 1: Electrical circuit for Question 1.

1a. Formulate the mathematical model of the circuit in the *state-space representation.* Consider the state variables $x_1 = i_{L_1}$, $x_2 = i_{L_2}$, and $x_3 = v_C$, the input $u = v_i$, and the output $y = v_o$ (the voltage across R). 10 Marks

Part b

Consider the state-space model for a system as:

$$\dot{\mathbf{x}}(t) = A\,\mathbf{x}(t) + B\,u(t), \qquad y(t) = C\,\mathbf{x}(t) + D\,u(t),$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

1b.1 Obtain the transfer function between the output y and the input u using the state-space method. 6 Marks

1b.2 Sketch the location of the poles and zeros in the s-plane and discuss the stability of the system. 4 Marks

Question 2

15 Marks

Consider the block diagram shown in Figure 2. The closed-loop system represents a satellite attitude control system with both angular position and angular velocity feedback. The constant gains K and K_v are to be determined.

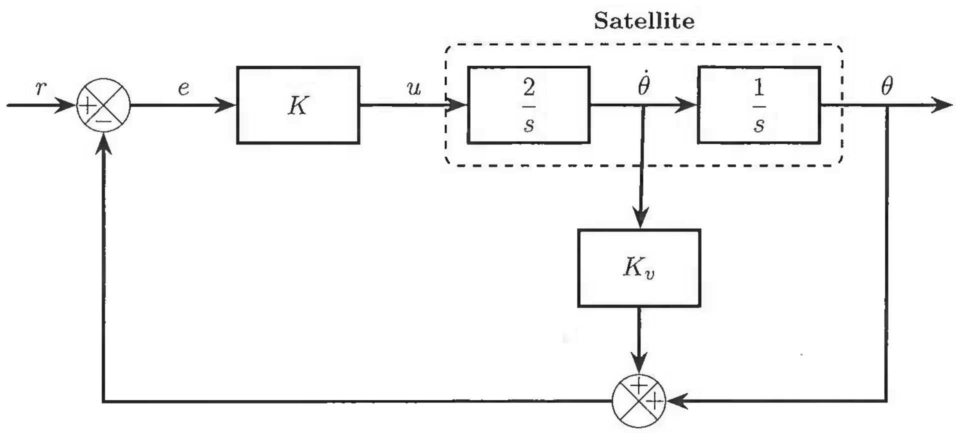

Figure 2: Block diagram of the satellite control system.

2.1. Simplify the block diagram and determine the transfer function $G(s)$ between the input $r(t)$ and the output $\theta(t)$. **10 Marks**

2.2. Compute the steady-state error e_{ss} of the closed-loop system for a unit-step input. **5 Marks**

Question 3

20 Marks

Consider the block diagram shown in Figure 3.

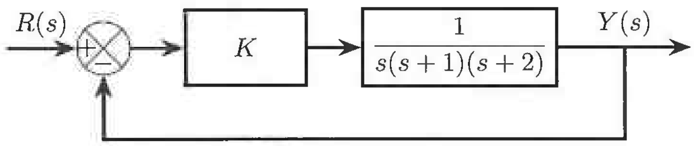

Figure 3: Block diagram for Question 3.

3.1. Using the *Routh–Hurwitz stability criterion*, determine the range of values of the gain K for which the closed-loop system remains stable. 5 Marks

3.2. Sketch the *root locus* of the system Clearly indicate the asymptotes, breakaway points, and imaginary-axis crossings (if any). 10 Marks

3.3. Determine the value of the gain K that makes the closed-loop system *marginally stable.* For this value of K, calculate the corresponding *natural frequency* of oscillation. 5 Marks

Question 4

15 Marks

The open-loop transfer function of a third-order system is given by:

$$L(s) = \frac{80(s+1)}{s(s+4)(s+20)}.$$

4.1. Sketch the magnitude and phase Bode plot of the open-loop transfer function. Use logarithmic scales and apply asymptotical approximations. 10 Marks

4.2. Calculate and show the gain and phase margin for the transfer function $L(s)$ using the Bode plot. 5 Marks

Key Formulas

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$	$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$	$T_s = \dfrac{4}{\zeta\omega_n}$
$T_r = \dfrac{1.76\zeta^3 + 0.417\zeta^2 + 1.039\zeta + 1}{\omega_n}$	$\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\frac{\%OS}{100}\right)\right)^2}}$	$1 + G(s)H(s) = 0$
$\lvert G(s)H(s)\rvert = 1$	$\angle G(s)H(s) = (2k+1)180^\circ$	$\sigma_A = \dfrac{\sum \text{poles} - \sum \text{zeros}}{n-m}$
$\theta_A = \dfrac{(2k+1)180}{n-m}$	$\dfrac{dK}{ds} = 0 \quad \text{where } K = -\dfrac{D(s)}{N(s)}$	$K = -\dfrac{D(s_0)}{N(s_0)}$
$G(s) = C(sI-A)^{-1}B + D$	$\det(sI - A) = 0$	

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$e^{at}\cos(\omega t)$	$\frac{s-a}{(s-a)^2+\omega^2}$
$e^{at}\sin(\omega t)$	$\frac{\omega}{(s-a)^2+\omega^2}$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$

Laplace Transform Properties
$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s)$
$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$
$\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$
$\mathcal{L}\{f'(t)\} = sF(s) - f(0^+)$
$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$
$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$
$\mathcal{L}\{f * g\} = F(s)G(s)$
$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$
$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

END OF PAPER